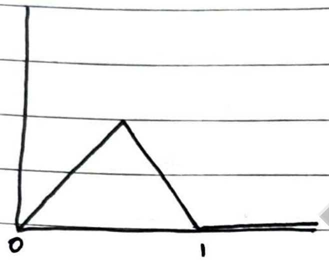
Continuous Random Variable.

- A graph f(x) representing continuous random variables is called probability density function.
- it has following properties
  - Total area under the graph = 1
  - it can't be negative



$y = f(x), \quad 0 \le x \le 1$

$= 0, \quad$ otherwise.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Q. $f(x) = kx^4 + \frac{1}{5}, \quad 0 \le x \le 2$

$= 0, \quad$ otherwise

a) find k

$$\int_0^2 kx^4 + \frac{1}{5}$$

$$\left[\frac{kx^5}{5} + \frac{1}{5}x\right]_0^2 = 1$$

$$\left[\frac{32k}{5} + \frac{2}{5}\right] - 0 = 1$$

$$\frac{32k + 2}{5} = 1$$

$$32k = 3$$

$$\boxed{k = \frac{3}{32}}$$

b) find $P(0 \le X \le 1)$

$$\int_0^1 \frac{3}{32}x^4 + \frac{1}{5}$$

$$\frac{3}{32} \times \frac{x^5}{5} + \frac{1}{5}$$

$$\frac{3}{32} \times \frac{1}{5} + \frac{1}{5} = \frac{7}{32}$$

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Q] $f(x) = \frac{x^3}{60}$, $2 \le x \le 4$

$= 0$, otherwise

a) find $P(x > 3)$

$$\int_3^4 \frac{x^3}{60} \qquad \left[\frac{x^4}{4 \times 60}\right]_3^4 \qquad \frac{4^4}{240} - \frac{3^4}{240}$$

$$= \frac{35}{48} , \quad \underline{0.73}$$

⇒ The median 'm' of a continuous random variable is the value such that. $P(X < m) = \int_{-\infty}^{m} f(x) \cdot dx = \frac{1}{2}$

Q. $f(x) = \begin{cases} k(x-2) & 2 \le x \le 5 \\ k(7-x) & 5 \le x \le 7 \\ 0 & \text{otherwise} \end{cases}$

i] show that $k = \frac{2}{13}$

$$\int_2^5 k(x-2) + \int_5^7 k(7-x) = 1$$

$$\int_2^5 kx - 2k + \int_5^7 7k - kx = 1$$

$$\left[\frac{kx^2}{2} - 2kx\right]_2^5 + \left[7kx - \frac{kx^2}{2}\right]_5^7 = 1$$

$$\left[\frac{25k}{2} - 10k\right] - \left[\frac{4k}{2} - 4k\right] + \left[49k - \frac{49k}{2}\right] - \left[35k - \frac{25k}{2}\right]$$

$$\frac{13}{2}k = 1$$

$$\boxed{k = \frac{2}{13}}$$

⇒ Expected value and variance of a continuous random variable

$$E(x) = \int_{-\infty}^{+\infty} x \, f(x) \, . dx$$

$$var(x) = \int_{-\infty}^{+\infty} x^2 \, f(x) \cdot dx - \left[E(x)\right]^2$$

Q. $f(x) = \frac{6}{125}\left[5x - x^2\right] \quad 0 \le x \le 5$

$= 0 \quad$ otherwise

$$E(x) = \int_0^5 \frac{6}{25}x^2 - \frac{6x^3}{125}$$

$$= \int_0^5 \frac{6x^3}{75} - \frac{6x^4}{500}$$

$$= 2.5$$

Q. $f(x) = \begin{cases} kx^2 & 0 \le x \le 2 \\ k(6-x) & 2 \le x \le 6 \\ 0 & \end{cases}$

$$\int_0^2 kx^2 + \int_2^6 k(6-x)$$

$$\int_0^2 \frac{kx^3}{3} + \int_2^6 6kx - \frac{kx^2}{2} = 1$$

$$k\int_0^2 \frac{x^3}{3} + k\int_2^6 6x - \frac{x^2}{2} = 1$$

$$\frac{8}{3} + \left[18 - 10\right] = \frac{1}{k}$$

$$\frac{32}{3} = \frac{1}{k} \qquad \boxed{k = \frac{3}{32}}$$

find mean and var(x)

$$E(x) = \int_0^2 x\, f(x).dx + \int_2^6 x\, f(x).dx$$

$$= \int_0^2 x \times kx^2 + \int_2^6 x \times k(6-x)$$

$$= \int_0^2 \frac{kx^4}{4} + \int_2^6 \frac{6kx^2}{2} - \frac{kx^3}{3}$$

$$\frac{3}{32} \times \frac{2^4}{4} + \left[\frac{27}{8} - \frac{1}{8}\right]$$

$$\frac{3}{8} + \frac{27}{8} - \frac{7}{8} = \boxed{\frac{23}{8}}$$